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HEAT ANALOGY IN LANGMUIR PROBE THEORY

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Mathematical equivalence of the problems of determining the Langmuir probe saturation current and the Nusselt number is shown in the heat transfer of a body of the same shape. Numerical results of the solution to these two problems obtained in various works are compared for a cylinder.

To determine the concentration of charged particles in a dense plasma from the volt-ampere characteristic of a probe, use is usually made of the ion saturation current. This is some limiting current at the probe with the ratio of the Debye screening distance to the probe radius $\alpha = \lambda_D/R \to 0$ and a sufficiently high negative potential when the probe current is due to positive ions alone.

To theoretically determine the saturation current, we need to solve one equation in partial derivatives, in place of a system of nonlinear elliptic equations that describe the total probe characteristic [1]. In the case of a weakly ionized incompressible isothermal plasma made up of positive monovalent ions and electrons and frozen chemical reactions in a plasma flow, it is the convective mass transfer equation. We write it in a dimensionless form

$$\frac{1}{2}\,\mathrm{Re}\,\mathrm{Sc}\,(u\nabla n) - \Delta n = 0\,, \tag{1}$$

where $\mathrm{Sc} = \nu/D_i$ is the Schmidt number. The factor 1/2 is due to the fact that the diffusion of the charged particles at the probe is ambipolar.

In the case of a cylindrical probe crossing the incoming flow and a spherical probe, the boundary conditions for Eq. (1) are

$$n\,|_{r=1} = 0\,, \quad n\,|_{r\to\infty} \to 1\,,$$

The dimensionless density of the saturation current j is given by the expression [1]:

$$j = 2\,\frac{\partial n}{\partial r}\bigg|_{r=1}\,.$$

The local Nusselt number determined from the diameter of the cylindrical or spherical body

$$\mathrm{Nu}_D = 2\,\frac{\partial T}{\partial r}\bigg|_{r=1}\,,$$

in the problem of convective heat transfer for an incompressible viscous liquid with constant physical characteristics is found, as is well known, from the solution to the equation

$$\mathrm{Re}\,\mathrm{Pr}\,(u\nabla T) - \Delta T = 0\,,$$

where T is the dimensionless temperature with the boundary conditions

$$T\,|_{r=1} = 0\,, \quad T\,|_{r\to\infty} \to 1\,.$$

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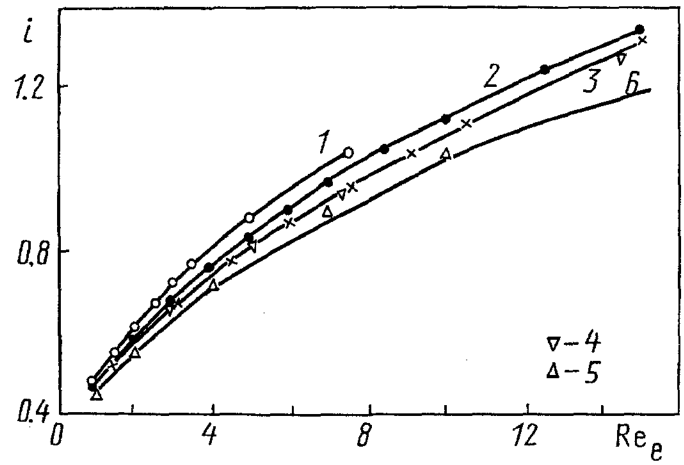

Fig. 1. Dimensionless saturation current onto a cylindrical probe vs electrical Reynolds number. Curves 1-3 are the results of [5]: 1) Sc = 0.5; 2) 1; 3) 1.5; 4 and 5) recalculated results of [6]; 4) Pr = 0.73; 5) 1; 6) recalculated result of [9].

As we can see, the indicated two problems become totally identical when the condition

$$\mathrm{Sc} = 2\,\mathrm{Pr}. \tag{2}$$

is satisfied.

Between the Reynolds electric number $\mathrm{Re}_e = \mathrm{ReSc}$ and the Peclet number $\mathrm{Pe} = \mathrm{RePr}$ there is the relationship

$$\mathrm{Re}_e = 2\,\mathrm{Pe}. \tag{3}$$

The considered heat analogy has been known in Langmuir probe theory and even earlier. Apparently, the first mention of it was made in [2]. In [3], heat analogy was used to determine the saturation current density in the critical point of a cylindrical probe. However, this analogy failed to find proper reflection in the literature.

Thus, it is only the limiting cases of convection, i.e., its weak $\mathrm{Re}_e \ll 1$ and strong $\mathrm{Re}_e \gg 1$ influence, that were considered in the known work [4] for a probe in the regime of continuum. The case of moderate influence of convection in which $\mathrm{Re}_e = 0$ (1) is numerically investigated in [5] for cylindrical and spherical probes. However, much earlier in the past, works [6, 7] were available in which the heat transfer of a cylinder in a viscous liquid was investigated for the same Re numbers as in [5]. Had the heat analogy been well known, the calculations of [5] would have probably been unnecessary, at least for a cylindrical probe.

The aim of the present work is to compare the results [5-7] pertaining to various physical phenomena yet actually obtained in solving the same mathematical problem.

For the cylindrical probe, the results [5] of calculating the dimensionless integral saturation current i, in which the gradient of charged particle concentration, averaged over the outline of the cylinder, was determined from the radius, are presented in Fig. 1 as functions of the number Re_e. It is necessary that the comparison of these results be performed with the average Nusselt number Nu determined also from the radius.

The values of Nu numbers obtained in [6, 7] are in good agreement. For comparison with the saturation current, it is more convenient to use [6] in which the calculated values of the number Nu are shown in tabulated form. This enables us to easily plot these values on conversion from the number Pe to the number Re_e by relation (3).

As the figure shows, the results of calculating the saturation current at Sc = 1.5 are very close to the results [6] at Pr = 0.73. The number Nu as a function of Re_e at Pr = 1 can be considered as an extrapolation of the results [5] obtained in the range of Sc = 0.5–1.5 to the number Sc = 2. All this confirms the reliability of the results of [5]. In [5], an equation is proposed that approximates the results of the calculations at Sc = 1:

$$i = 0.43\,\mathrm{Re}_e^{0.42}. \tag{4}$$

The dependence of i on Sc at fixed Re_e is weak.

The dimensional current is related to the dimensionless one by

$$I = 4\pi e N_\infty D_i L i. \tag{5}$$

The applicability of Eq. (4) is limited by $\mathrm{Re}_e \le 15$. With larger Re numbers, complicated transient and separation phenomena occur at the back surface of the cylinder. Calculating the saturation current under these conditions is possible (the problem of convective heat transfer is solved, for example, in [8]) yet involves difficulties. Therefore the proposal of [3] to use experimental data on the cylinder heat transfer for plasma diagnostics with Re > 15 seems reasonable.

In [9], a unified equation is presented that relates the heat transfer from a circular cylinder in transverse flow of the air in the $5 \cdot 10^{-3}$ to $1 \cdot 10^5$ Re number range (Re is determined from the radius). This equation is the approximation of experimental data. For the saturation current, in view of (2) and (3), we can rewrite it as

$$i = 0.092 + 0.190\,\mathrm{Re}_e^{0.5} + 0.146\,(\mathrm{Re}_e/0.73)^{\chi}, \tag{6}$$

where $\chi = 0.247 + 0.0476\mathrm{Re}_e^{0.168}$. This equation is obtained at Pr = 0.73 which corresponds to Sc = 1.46.

The dependence (6) is also given in Fig. 1. The values of saturation currents determined using (6) are found to be somewhat smaller than those proposed in [5-7]; however, the disagreement does not exceed 10%.

We can determine the concentration of the charged particles in the plasma by the ion saturation current of the cylindrical probe from relation (5); in this case, with $\mathrm{Re}_e \le 15$, expression (4) would be appropriate for use for i.

For the saturation current at the spherical probe at Sc = 1 in [5]

$$I = 8\pi e N_\infty D_i R i,$$

is obtained where

$$i = 1 + 0.2\,\mathrm{Re}_e^{0.62}$$

with $\mathrm{Re}_e \le 65$.

By using the heat analogy we can also perform the opposite, having obtained for the heat transfer of the sphere in forced convection

$$\mathrm{Nu} = 1 + 0.3\,\mathrm{Pe}^{0.62}$$

at Pr = 0.5.

We propose to use the equations given in the present work in the diagnostics of a laminar incompressible plasma.

NOTATION

Re, Re_e, gasdynamical and electrical Reynolds numbers; Pr, Prandtl number; Pe, Peclet number; Nu, average Nusselt number; Nu_D, local Nusselt number determined from the diameter; α, Debye screening distance to the probe radius ratio; λ_D, Debye screening distance; R, probe radius; u, velocity field of a neutral gas; ν, kinematic viscosity factor; D_i, ion diffusion coefficient; n, dimensionless concentration of charged particles; r, radial coordinate; j, dimensionless density of the saturation current; i, dimensional integral saturation current; I, dimensional saturation current; e, electron charge; N_∞, concentration of charged particles in the incoming flow; L, probe length.

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